



Rossmoyne Senior High School

Semester One Examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

WA student number: In figures

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Circle your teacher:

Ms Alvaro	Ms Lee
Mr Liu	Ms Robinson

In words

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Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	93	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	8	
9	6	
10	7	
11	6	
12	8	
13	8	
14	8	
15	10	
16	7	
17	8	
18	9	
19	8	
Section 2 Total	93	
Weighted ($\times 0.6915$)	65%	

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Section Two: Calculator-assumed

65% (93 Marks)

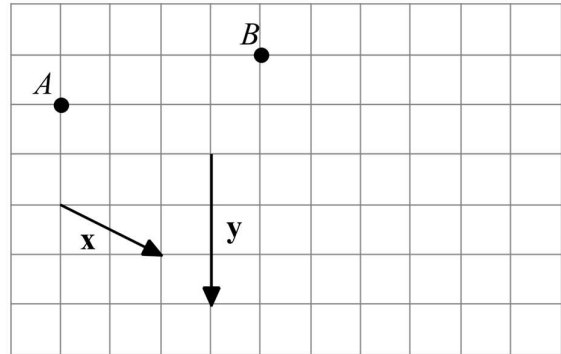
This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8

(8 marks)

Two points, A and B , as well as two vectors, $\vec{x}$ and $\vec{y}$, are shown on the grid on the right.



- (a) Write down a vector in terms of $\vec{x}$ and $\vec{y}$ that gives $\overrightarrow{AB}$. (2 marks)

- (b) (i) It is given that $\overrightarrow{AC} = 3\vec{x}$. Show the position of point C on the above grid. (1 mark)

- (ii) Hence, determine a vector in terms of $\vec{x}$ and $\vec{y}$ that gives $\overrightarrow{BC}$. (2 marks)

The vector $\vec{p}$ is the vector projection of $\vec{x}$ onto $\vec{y}$.

- (c) (i) Draw, and clearly label, vector $\vec{p}$ on the above grid. (1 mark)

- (ii) State $\vec{p}$ in terms of $\vec{x}$ and/or $\vec{y}$. (1 mark)

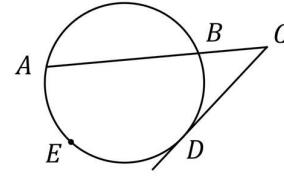
- (iii) Hence, determine the vector projection of $4\vec{x}$ onto $2\vec{y}$ in terms of $\vec{p}$. (1 mark)

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Question 9

(6 marks)

- (a) In the diagram (not drawn to scale), the tangent to the circle at D meets the extension of chord AB at C , and E lies on the arc AD , where $AEDB$ is a major segment of the circle. The radius of the circle is 3.3 cm, $AB = 4.5$ cm and $CD = 5.4$ cm.



- (i) Determine the length BC .

(2 marks)

- (ii) Determine, to the nearest degree, the size of $\angle AEB$.

(2 marks)

- (b) The points P, Q, R, S and T lie in that order on the circumference of a circle, so that $PQ = PS$ and $\angle PRQ = 37^\circ$. Determine, with justification, the size of $\angle PTS$. (2 marks)

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Question 10**(7 marks)**

- (a) Anders is the recruitment manager for a building company and has just interviewed 85 candidates, of whom 44 were qualified carpenters, 52 were qualified electricians and 37 were qualified plumbers. Furthermore, 11 of the candidates were qualified in all three trades and all of them were qualified in at least one trade. Determine the number of candidates Anders hired, given that he hired all candidates qualified in at least two trades.

(3 marks)

- (b) Determine how many of the integers between 1 and 2024 inclusive are

(i) divisible by 42.

(1 mark)

(ii) divisible by 14 or 21 but not both.

(3 marks)

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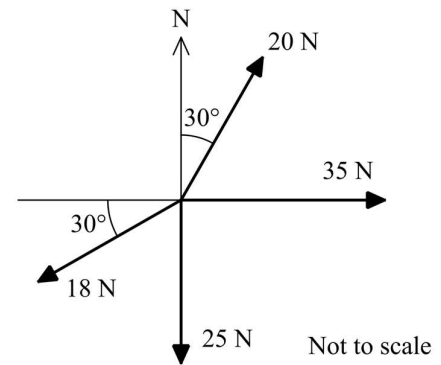
Question 11

(6 marks)

The diagram on the right shows four forces acting on a body.

All the forces are in the same plane.

- (a) Find the magnitude and bearing of the resultant, *correct to two decimal places*. (4 marks)



- (b) Find the magnitude and bearing of the single force that will keep this system in equilibrium, *correct to two decimal places*. (2 marks)

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Question 12

(8 marks)

The heights of players in a competitive basketball squad are categorised as tall, medium or short, and of the 16 players in the squad, 6 are tall, 7 medium and the rest short.

- (a) When players are randomly selected from the squad, determine, the least number of players that must be picked to be certain that at least 5 players with the same height category are chosen. (2 marks)

- (b) Determine the number of distinct ways that 5 players can be chosen from the squad

- (i) with no restrictions. (1 mark)

- (ii) so that exactly one player is short. (1 mark)

- (iii) so that at least half of the players are tall. (2 marks)

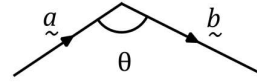
- (c) The name of the coach of the squad is ANNEMARIEKA. Determine the number of distinct arrangements of all 11 letters in her name with the vowels and consonants alternating. (2 marks)

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Question 13

(8 marks)

- (a) Determine the exact scalar product of the two vectors shown (not to scale), given that $\theta = 150^\circ$, $|\vec{a}| = 6$ and $|\vec{b}| = 15$.



(2 marks)

- (b) Let $\vec{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\vec{s} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ and $\vec{t} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$, where $\vec{t} = \lambda\vec{r} + \mu\vec{s}$. Determine

(i) $4\vec{r} - 7\vec{s} + 3\vec{t}$.

(1 mark)

- (ii) the angle between the directions of $\vec{r}$ and $\vec{s}$.

(2 marks)

- (iii) the value of the constant λ and the value of the constant μ .

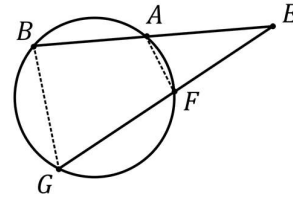
(3 marks)

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Question 14

(8 marks)

The diagram shows two secants drawn through the same circle from point E that is external to the circle.



One secant from point E meets the circle first at point A and then at point B and the other secant meets the circle first at point F and then at point G .

- (a) Prove that triangle AEF is similar to triangle EGB .

(4 marks)

- (b) Show that $EA \cdot EB = EF \cdot EG$.

(1 mark)

- (c) Determine all possible lengths of EB when $AB = 34$ cm, EF is 4 cm longer than AE , and $FG = EF$.

(3 marks)

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Question 15

(10 marks)

$\underline{\underline{a}}$ and $\underline{\underline{b}}$ are two vectors such that $|\underline{\underline{a}}| = 2$ and $|\underline{\underline{b}}| = 3$. The angle between $\underline{\underline{a}}$ and $\underline{\underline{b}}$ is $\frac{\pi}{3}$.

Two other vectors are defined as $\underline{\underline{u}} = \lambda \underline{\underline{a}} + \underline{\underline{b}}$ and $\underline{\underline{v}} = \underline{\underline{a}} + \lambda \underline{\underline{b}}$, such that $\lambda > 0$.

(a) Show that $|\underline{\underline{u}}| = \sqrt{4\lambda^2 + 6\lambda + 9}$. (3 marks)

(b) Determine the expression for $|\underline{\underline{v}}|$, similar to the form in Part (a). (3 marks)

(c) Determine the value(s) of λ , *correct to 2 decimal places*, such that the angle between $\underline{\underline{u}}$ and $\underline{\underline{v}}$ is $\frac{\pi}{6}$. (4 marks)

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Question 16

(7 marks)

- (a) A plain pink shirt has five buttons, each of which must be one of seven different colours. Determine the number of different button arrangements on the shirt if repetition of colour

(i) is allowed. (1 mark)

(ii) is not allowed. (1 mark)

- (b) When n objects arranged in a row are rearranged so that none of them occupy their original positions, the objects are said to be deranged. For example, BCA is a derangement of ABC but BAC is not because the C occupies its original position. The number of derangements D_n of n distinct objects is shown in the table below.

n	4	5	6	7	8	9
D_n	9	44	265	1854	14833	133496

- (i) List all the permutations of the letters A, B and C , circle the derangements of ABC and hence state the value of D_3 . (2 marks)

- (ii) Given 10 people and their passport photos, determine the number of ways in which the people can be matched with the photos so that exactly 4 of them have the correct photo. (3 marks)

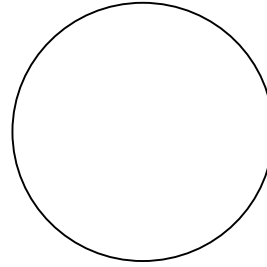
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Question 17**(8 marks)**

PQ and RS are two intersecting chords in the same circle and RS is parallel to the tangent to the circle at P .

- (a) Sketch a diagram to show this information.

(1 mark)



- (b) Prove that PQ bisects $\angle RQS$.

(4 marks)

- (c) In the case when RS is a diameter, the acute angle between PQ and RS is 67° and the length of minor arc QR is less than the length of minor arc QS , determine the size of $\angle QOR$, where O is the centre of the circle. Justify your answer. (3 marks)

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Question 18

(9 marks)

The navigator of a small pilot boat with a cruising speed of 12.5 km/h must calculate the required constant velocity vector $\begin{pmatrix} x \\ y \end{pmatrix}$ km/h the boat should steer from port P to ship S . Relative to a nearby lighthouse L , $\overrightarrow{LP} = \begin{pmatrix} 0.91 \\ -1.22 \end{pmatrix}$ km and $\overrightarrow{LS} = \begin{pmatrix} 2.19 \\ 4.08 \end{pmatrix}$ km, and a steady current of $\begin{pmatrix} -1.20 \\ 1.55 \end{pmatrix}$ km/h flows in the local waters.

(a) Explain why $x^2 + y^2 = 156.25$. (1 mark)

(b) Explain why $\begin{pmatrix} x - 1.2 \\ y + 1.55 \end{pmatrix} = \lambda \begin{pmatrix} 1.28 \\ 5.3 \end{pmatrix}$, where λ is a constant, and hence deduce another relationship between x and y . (4 marks)

(c) Using CAS, or otherwise, determine the required velocity vector $\begin{pmatrix} x \\ y \end{pmatrix}$ for the boat. (2 marks)

(d) At what time, to the nearest minute, would the pilot boat reach ship S if it left port P at 7:48 am? (2 marks)

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Question 19**(8 marks)**

Relative to the origin O , points P and Q have position vectors $\vec{p}$ and $\vec{q}$ respectively. Point A lies on OP so that $\vec{OA} = k\vec{p}$, point B lies on OQ so that $\vec{OB} = (1 - k)\vec{q}$ and point R lies on AB so that $\vec{AR} = (1 - k)\vec{AB}$, for some scalar constant k where $0 < k < 1$.

- (a) Sketch a diagram to show O and the other five points. (1 mark)

- (b) Show that $\vec{OR} = k^2\vec{p} + (1 - k)^2\vec{q}$. (3 marks)

Let d be the distance of R from the origin when $\vec{p} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

- (c) Show that $d = \sqrt{9(1 - k)^4 + 16k^4}$. (2 marks)

- (d) By graphing d against k , or otherwise, determine the minimum value of d and the value of k required to achieve this minimum, giving your answers *correct to the nearest* 0.001. (2 marks)

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Supplementary page

Question number: _____

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